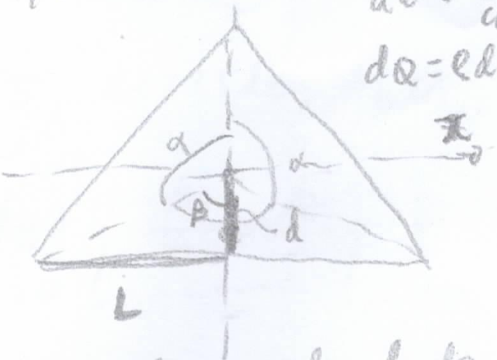
1NORMAL/2025

$P = 600\,mm$

$\rho = 10\,nC/m$

$\varepsilon = \varepsilon_0$

$z = 20\,mm$



$d\vec{E} = \dfrac{dQ}{4\pi\varepsilon R^2}\vec{a}_r$

$dQ = \rho\, dl$

$\begin{cases} 3\alpha = 360^\circ \\ \alpha = 2\beta \end{cases} \Rightarrow \beta = 60^\circ$

$6L = 600 \Rightarrow L = 100\,mm = 0{,}1\,m$

$tg\,\beta = \dfrac{L}{d} \Rightarrow tg\,60 = \dfrac{100}{d} \Rightarrow d \simeq 57{,}74\,mm = 0{,}05774\,m$

devido à simetria cada um dos lados do triângulo contribui da mesma forma para o campo segundo z e as componentes segundo x e y anulam-se. Logo o campo resultante

$\vec{E} = 3\vec{E}_{1z}$ — é a componente gerada por um dos lados do triângulo segundo z e multiplicado por 3

$\vec{r}_i = (x, -d, 0)$

$\vec{r}_f = (0, 0, z)$

$$R = |\vec{r}_f - \vec{r}_i| = |(0,0,z) - (x,-d,0)| = \sqrt{x^2 + d^2 + z^2}$$

$$\vec{a}_R = \frac{\vec{r}_f - \vec{r}_i}{R} = \frac{(-x, d, z)}{\sqrt{x^2 + d^2 + z^2}}$$

$$d\vec{E}_{1z} = \frac{\rho\, dx}{4\pi\varepsilon_0 R^2} \cdot \frac{z}{R}\vec{u}_z = \frac{\rho\, dx\, z}{4\pi\varepsilon_0 R^3}\vec{u}_z$$

$$\vec{E}_{1z} = \frac{\rho z}{4\pi\varepsilon_0}\vec{u}_z \int_{-L}^{L} \frac{dx}{\left[x^2 + \left(\sqrt{d^2+z^2}\right)^2\right]^{3/2}} =$$

$$= \frac{\rho z \vec{u}_z}{4\pi\varepsilon_0 (d^2+z^2)} \left[\frac{x}{\sqrt{x^2 + d^2 + z^2}}\right]_{-L}^{L} =$$

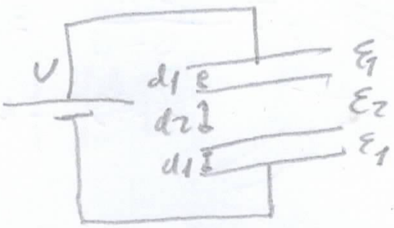
$$= \frac{\rho z \vec{u}_z}{4\pi\varepsilon_0} \cdot \frac{2L}{(d^2+z^2)\sqrt{L^2 + d^2 + z^2}} =$$

$$= \frac{10\times10^{-9} \times 20\times10^{-3} \times \vec{u}_z}{4\pi \times 8{,}854\times10^{-12}} \cdot \frac{2\times0{,}1}{\left[(57{,}74\times10^{-3})^2 + (20\times10^{-3})^2\right]\sqrt{0{,}1^2 + (57{,}74\times10^{-3})^2 + (20\times10^{-3})^2}}$$

$$\frac{359{,}5\times10^{-3}\,\vec{u}_z}{3{,}73\times10^{-3} \times 117\times10^{-3}} \simeq 821{,}6\,V/m$$

$$\vec{E}_T = 3 \times \vec{E}_{1z} = 3 \times 823{,}8 = 2{,}46\,kV/m$$

$\varepsilon_{r1}/\varepsilon_{r2} = 5$, $d_1 = 25 d_2$, A, U, $D = \varepsilon E$



$$\begin{cases} \phi_1 = \phi_2 \\ 2U_1 + U_2 = U \end{cases} \quad \begin{cases} \dfrac{\phi_1}{A} = \dfrac{\phi_2}{A} \\ \_\_\_ \end{cases} \quad \begin{cases} D_1 = D_2 \\ \_\_\_ \end{cases}$$

$$\begin{cases} \varepsilon_1 E_1 = \varepsilon_2 E_2 \\ \_\_\_ \end{cases} \quad \begin{cases} \varepsilon_1 \dfrac{U_1}{d_1} = \varepsilon_2 \dfrac{U_2}{d_2} \\ \_\_\_ \end{cases} \quad \begin{cases} \dfrac{\varepsilon_0 \varepsilon_{r1}}{\varepsilon_0 \varepsilon_{r2}} U_1 = U_2 \dfrac{d_1}{d_2} \\ 2U_1 + U_2 = U \end{cases}$$

$$\begin{cases} 5U_1 = 25 U_2 \\ 2U_1 + U_2 = U \end{cases} \quad \begin{cases} U_1 - 5U_2 = 0 \\ 2U_1 + U_2 = U \end{cases} \quad \begin{cases} U_1 = 5U/11 \\ U_2 = U/11 \end{cases} \quad //$$

III

N, μ_r, L, A $\quad L = \dfrac{N\phi}{I}$,

$$L = \frac{N\phi}{I} \Rightarrow L = \frac{NBA}{I} \Rightarrow$$

$$L = \frac{N\mu_0 H_0 A}{I} \Rightarrow$$

$$H_0 = \frac{LI}{N\mu_0 A}$$

$$\frac{LI}{N\mu_0 A} = H_0 \quad //$$

IV

$$\vec{E} = \vec{E}_{max} \operatorname{sen}(\omega t - \beta z)\, \vec{a}_y$$

2 a) $|\eta| = z_0 \simeq 376{,}73\ \Omega$

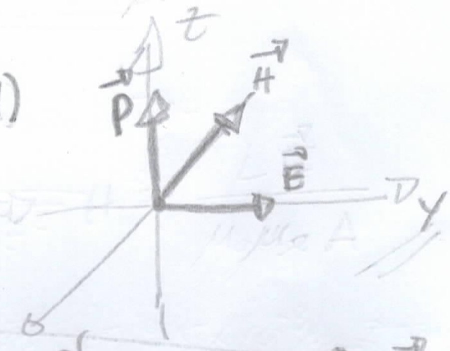
$$\frac{|\vec{E}|}{z_0} = |\vec{H}|$$

$$\vec{H} = -\frac{\vec{E}_{max} \operatorname{sen}(\omega t - \beta z)}{z_0} \vec{u}_x \quad //$$

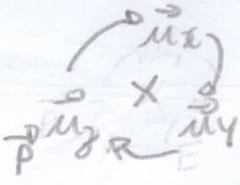
b)

$$P_{med} = \frac{|E_{max}|^2/2}{z_0} \quad //$$

1)



como $\vec{P} = \vec{E} \times \vec{H}$



$\vec{u}_z = \vec{u}_y \times \vec{u}_H$

então $\vec{u}_H = -\vec{u}_x$ //

$S = 250\ cm^2$

$$v = -N\frac{d\phi}{dt} = -NA\frac{dB}{dt} \Rightarrow |v| = NA\left|\frac{dB}{dt}\right|$$

$$5 = \left| -N\, 250\times10^{-4} \cdot \frac{(-5-8)\times10^{-3}}{(33-23)\times10^{-3}} \right| \Rightarrow N = \frac{5\times10^{4}}{325}$$

$N \simeq 154$ espiras