FORMULÁRIO DE ESTATÍSTICA

ISEP - DMA

2024/25

Formulário de Estatística

Medidas descritivas duma amostra

$$n=\sum_{i=1}^{c} n_i$$

$$\bar{x}=\frac{1}{n}\sum_{k=1}^{n} x_k=\frac{1}{n}\sum_{i=1}^{c} x_i n_i=\sum_{i=1}^{c} x_i f_i$$

$$Me=\begin{cases}\dfrac{x_{\frac{n}{2}}+x_{\frac{n}{2}+1}}{2} & , n\ \ par\\ x_{\frac{n+1}{2}} & , n\ impar\end{cases}$$

$$z_\alpha=L_k+\frac{h_k}{f_k}\left(\alpha-F_{k-1}\right)$$

$$s^2=\frac{1}{n-1}\sum_{k=1}^{n}\left(x_k-\bar{x}\right)^2=\frac{1}{n-1}\left[\sum_{k=1}^{n} x_k^2-n\bar{x}^2\right]=$$

$$=\frac{n\sum_{k=1}^{n} x_k^2-\left(\sum_{k=1}^{n} x_k\right)^2}{n(n-1)}$$

$$s^2=\frac{1}{n-1}\sum_{i=1}^{c}\left(x_i-\bar{x}\right)^2 n_i=\frac{1}{n-1}\left[\sum_{i=1}^{c} x_i^2 n_i-n\bar{x}^2\right]=$$

$$=\frac{n\sum_{i=1}^{c} x_i^2 n_i-\left(\sum_{i=1}^{c} x_i n_i\right)^2}{n(n-1)}$$

$$CV=\frac{s}{\left|\bar{x}\right|}$$

$$m_r=\frac{1}{n}\sum_{k=1}^{n}\left(x_k-\bar{x}\right)^r=\frac{1}{n}\sum_{i=1}^{c}\left(x_k-\bar{x}\right)^r n_i$$

$$a_r=\frac{m_r}{s^r}$$

Regressão linear

$$y = a + bx$$

$$b = \frac{S_{xy}}{S_{xx}}$$

$$a = \bar{y} - b\bar{x}$$

$$S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

$$S_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$$

$$S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$

$$sqt = \sum_{i=1}^{n}(y_i - \bar{y})^2 = S_{yy}$$

$$sqe = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}[y_i - y(x_i)]^2 = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = S_{yy} - bS_{xy}$$

$$sqr = \sum_{i=1}^{n}[y(x_i) - \bar{y}]^2 = sqt - sqe = \frac{S_{xy}^2}{S_{xx}} = bS_{xy}$$

$$r^2 = \frac{sqr}{sqt} = 1 - \frac{sqe}{sqt} = \frac{S_{xy}^2}{S_{xx}S_{yy}} = b^2\frac{S_{xx}}{S_{yy}}$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = b\sqrt{\frac{S_{xx}}{S_{yy}}}$$

Formulário de Estatística

Propriedades de conjuntos

$\overline{S} = \emptyset$

$\overline{\emptyset} = S$

$\overline{\overline{A}} = A$

$A \cap \overline{A} = \emptyset$

$A \cup \overline{A} = S$

$A \cup \emptyset = A$

$A \cap S = A$

$A \cup S = S$

$A \cap \emptyset = \emptyset$

$A \cap B = B \cap A$

$A \cup B = B \cup A$

$A \cap B \cap C = (B \cap C) = (A \cap B) \cap C$

$A \cup B \cup C = A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$A \setminus B = A \cap \overline{B}$

A e B são mutuamente exclusivos se $A \cap B = \emptyset$

Teoria das probabilidades

$0 \leq P(A) \leq 1$

$P(\emptyset) = 0$

$P(S) = 1$

$P(\overline{A}) = 1 - P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - \\ - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

A e B são acontecimentos independentes $\Leftrightarrow P(A \cap B) = P(A)P(B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

$$P(A) = \sum_{i=1}^{r} P(A|B_i)P(B_i)$$

$$P(B_k|A) = \frac{P(B_k \cap A)}{P(A)} = \frac{P(A|B_k)P(B_k)}{\sum_{i=1}^{r} P(A|B_i)P(B_i)}$$

Formulário de Estatística

Distribuições discretas

$$f(x) = P(X = x) = \begin{cases} 0 & , x \neq x_i \\ p_i & , x = x_i \end{cases}$$

$$f(x) \geq 0 \quad \wedge \quad \sum_{x_i} f(x_i) = 1$$

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

$$P(a < X \leq b) = F(b) - F(a)$$

$$\mu = E(X) = \sum_{x_i} x_i f(x_i)$$

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = E(X^2) - E^2(X) = \sum_{x_i} x_i^2 f(x_i) - \mu^2$$

Distribuições contínuas

$$P(a < X < b) = \int_a^b f(x)dx$$

$$f(x) \geq 0 \quad \wedge \quad \int_{-\infty}^{+\infty} f(x)dx = 1$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$$

$$P(a < X \leq b) = F(b) - F(a)$$

$$\mu = E(X) = \int_{-\infty}^{+\infty} xf(x)dx$$

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = E(X^2) - E^2(X) = \int_{-\infty}^{+\infty} x^2 f(x)dx - \mu^2$$

Propriedades de $E(X)$ e $V(X)$

$$E(a) = a$$

$$E(aX) = aE(X)$$

$$E(X \pm Y) = E(X) \pm E(Y)$$

$$E(XY) = E(X)E(Y) \quad \textit{se } X \textit{ e } Y \textit{ são v.a. independentes}$$

$$V(a) = 0$$

$$V(aX) = a^2 V(X)$$

$$V(X \pm Y) = V(X) + V(Y) \quad \textit{se } X \textit{ e } Y \textit{ são v.a. independentes}$$

Distribuição de Bernoulli

$$X \sim Bernoulli(p)$$

$$f(x) = \begin{cases} 0 & , x \neq 0,1 \\ p^x q^{1-x} & , x = 0,1 \end{cases}$$

$$F(x) = \sum_{x_i \leq x} p^{x_i} q^{1-x_i}$$

$$E(X) = p$$

$$V(X) = pq$$

Formulário de Estatística

Distribuição binomial

$X \sim Bi(n,p)$

$$f(x)=\begin{cases}0 & ,x\neq 0,1,2,\dots,n\\ C_x^n p^x q^{n-x} & ,x=0,1,2,\dots,n\end{cases}$$

$$F(x)=\sum_{x_i\leq x} C_{x_i}^n p^{x_i} q^{n-x_i}$$

$E(X)=np$

$V(X)=npq$

Distribuição de Poisson

$X \sim Po(\mu)$

$$f(x)=\begin{cases}0 & ,x\notin N_0\\ \dfrac{\mu^x}{x!}e^{-\mu} & ,x\in N_0\end{cases}$$

$$F(x)=\sum_{x_i\leq x}\frac{\mu^{x_i}}{x_i!}e^{-\mu}$$

$E(X)=\mu$

$V(X)=\mu$

Distribuição uniforme

$X \sim U(a,b)$

$$f(x)=\begin{cases}\dfrac{1}{b-a} & ,x\in[a,b]\\ 0 & ,x\notin[a,b]\end{cases}$$

$$F(x)=\begin{cases}0 & ,x<a\\ \dfrac{x-a}{b-a} & ,a\leq x\leq b\\ 1 & ,x>b\end{cases}$$

$$E(X)=\frac{a+b}{2}$$

$$V(X)=\frac{(b-a)^2}{12}$$

Distribuição exponencial

$X \sim Exp(\lambda)$

$$f(x)=\begin{cases}0 & ,x<0\\ \lambda e^{-\lambda x} & ,x\geq 0\end{cases}$$

$$F(x)=\begin{cases}0 & ,x<0\\ 1-e^{-\lambda x} & ,x\geq 0\end{cases}$$

$$E(X)=\frac{1}{\lambda}$$

$$V(X)=\frac{1}{\lambda^2}$$

Distribuição normal

$X \sim N(\mu,\sigma^2)$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$F(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}dt$$

$E(X)=\mu$

$V(X)=\sigma^2$

$$Z=\frac{X-\mu}{\sigma}\sim N(0,1)$$

$$P(X\leq a)=P\left(Z\leq\frac{a-\mu}{\sigma}\right)=\Phi\left(\frac{a-\mu}{\sigma}\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{a-\mu}{\sigma}}e^{-\frac{t^2}{2}}dt$$

Teorema da aditividade da normal

$$\left.\begin{array}{l}X_i\sim N(\mu_i,\sigma_i^2),\ i=1,2,\dots,n\\ X_1,X_2,\dots,X_n\ independentes\end{array}\right\}\Rightarrow\sum_{i=1}^{n}a_iX_i\sim N\left(\sum_{i=1}^{n}a_i\mu_i,\sum_{i=1}^{n}a_i^2\sigma_i^2\right)$$

$$\left.\begin{array}{l}X_i\sim N(\mu,\sigma^2),\ i=1,2,\dots,n\\ X_1,X_2,\dots,X_n\ independentes\end{array}\right\}\Rightarrow\sum_{i=1}^{n}X_i\sim N(n\mu,n\sigma^2)$$

Teorema do limite central (TLC)

$$\left.\begin{array}{l} X_1, X_2, \ldots, X_n \text{ independentes} \\ E(X_i) = \mu_i \wedge V(X_i) = \sigma_i^2,\ i = 1,2,\ldots,n \\ n \geq 30 \end{array}\right\} \Rightarrow \sum_{i=1}^{n} X_i \sim N\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$$

Amostragem

$$\left.\begin{array}{l} \{X_1, X_2, \ldots, X_n\} \text{ é amostra aleatória i.i.d.} \\ E(X_i) = \mu \wedge V(X_i) = \sigma^2 \\ n \geq 30 \vee X_i \sim N(\mu, \sigma^2) \end{array}\right\} \Rightarrow \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\left.\begin{array}{l} X \sim Bi(n, p) \\ n \geq 30 \end{array}\right\} \Rightarrow \hat{P} = \frac{X}{n} \sim N\left(p, \frac{pq}{n}\right)$$

Formulário de Estatística

Intervalo de confiança (I .C.) para μ

$$\mu = \overline{X} \pm \Delta$$

σ conhecido

$$\Delta = \lambda \frac{\sigma}{\sqrt{n}}$$

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

$$P(a \le \mu \le b) = \gamma \Leftrightarrow P(-\lambda \le Z \le \lambda) = \gamma$$

$$\left(n \ge 30 \ \vee \ X \sim \mathrm{N}(\mu, \sigma^2)\right) \Rightarrow Z \sim \mathrm{N}(0,1) \Rightarrow \Phi(\lambda) = \frac{1+\gamma}{2}$$

σ desconhecido

$$\Delta = \lambda \frac{S}{\sqrt{n}}$$

$$T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

$$P(a \le \mu \le b) = \gamma \Leftrightarrow P(-\lambda \le T \le \lambda) = \gamma$$

$$n \ge 30 \Rightarrow T \sim \mathrm{N}(0,1)$$

$$\text{Nota: } \quad n \ge 30 \Rightarrow \mathrm{T}(n-1) \approx \mathrm{N}(0,1)$$

$$\left(n < 30 \ \wedge \ X \sim \mathrm{N}(\mu, \sigma^2)\right) \Rightarrow T \sim \mathrm{T}(n-1)$$

I .C. para p

$$p = \widehat{P} \pm \Delta$$

$$\Delta = \lambda \sqrt{\frac{p(1-p)}{n}} \approx \lambda \sqrt{\frac{\widehat{P}(1-\widehat{P})}{n}}$$

$$Z = \frac{\widehat{P} - p}{\sqrt{\frac{\widehat{P}(1-\widehat{P})}{n}}}$$

$$P(a \le p \le b) = \gamma \Leftrightarrow P(-\lambda \le Z \le \lambda) = \gamma$$

$$n \ge 30 \Rightarrow Z \sim N(0,1) \Rightarrow \Phi(\lambda) = \frac{1+\gamma}{2}$$

I .C. para $p_1 - p_2$

$$p_1 - p_2 = \widehat{P}_1 - \widehat{P}_2 \pm \Delta$$

$$\Delta = \lambda \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} \approx \lambda \sqrt{\frac{\widehat{P}_1(1-\widehat{P}_1)}{n_1} + \frac{\widehat{P}_2(1-\widehat{P}_2)}{n_2}}$$

$$Z = \frac{(\widehat{P}_1 - \widehat{P}_2) - (p_1 - p_2)}{\sqrt{\frac{\widehat{P}_1(1-\widehat{P}_1)}{n_1} + \frac{\widehat{P}_2(1-\widehat{P}_2)}{n_2}}}$$

$$P(a \le p_1 - p_2 \le b) = \gamma \Leftrightarrow P(-\lambda \le Z \le \lambda) = \gamma$$

$$(n_1 \ge 30 \wedge n_2 \ge 30) \Rightarrow Z \sim \mathrm{N}(0,1) \Rightarrow \Phi(\lambda) = \frac{1+\gamma}{2}$$

I .C. para μ 1-μ 2

$$\mu_1 - \mu_2 = \overline{X}_1 - \overline{X}_2 \pm \Delta$$

σ1 e σ2 conhecidos

$$\Delta = \lambda \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$Z = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$P\left(a \le \mu_1 - \mu_2 \le b\right) = \gamma \Leftrightarrow P\left(-\lambda \le Z \le \lambda\right) = \gamma$$

$$\left[\begin{cases} n_1 \ge 30 \\ n_2 \ge 30 \end{cases} \vee \begin{cases} X_1 \sim \mathrm{N}\left(\mu_1, \sigma_1^2\right) \\ X_2 \sim \mathrm{N}\left(\mu_2, \sigma_2^2\right) \end{cases}\right] \Rightarrow Z \sim \mathrm{N}(0,1)$$

σ1 e σ2 desconhecidos

$$\Delta = \lambda \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$T = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

σ1 = σ2 =σ desconhecido

$$\Delta = \lambda S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \quad S^2 = \frac{\left(n_1 - 1\right)S_1^2 + \left(n_2 - 1\right)S_2^2}{n_1 + n_2 - 2}$$

$$T = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$P\left(a \le \mu_1 - \mu_2 \le b\right) = \gamma \Leftrightarrow P\left(-\lambda \le T \le \lambda\right) = \gamma$$

$$\begin{cases} n_1 \ge 30 \\ n_2 \ge 30 \end{cases} \Rightarrow T \sim \mathrm{N}(0,1)$$

$$\begin{cases} n_1 < 30 \vee n_2 < 30 \\ X_1 \sim \mathrm{N}\left(\mu_1, \sigma_1^2\right) \\ X_2 \sim \mathrm{N}\left(\mu_2, \sigma_2^2\right) \end{cases} \Rightarrow \begin{cases} T = \dfrac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim \mathrm{T}(m), \quad m \cong \dfrac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(S_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(S_2^2/n_2\right)^2}{n_2 - 1}} \\ T = \dfrac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim \mathrm{T}\left(n_1 + n_2 - 2\right) \end{cases}$$

Teste de hipóteses para μ

$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$

σ conhecido

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

$$\left(n \geq 30 \vee X \sim N\left(\mu, \sigma^2\right)\right) \Rightarrow Z \sim N(0,1)$$

σ desconhecido

$$T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

$$n \geq 30 \Rightarrow T \sim N(0,1) \quad ; \quad \left(n < 30 \quad \wedge \quad X \sim \mathrm{N}\left(\mu, \sigma^2\right)\right) \quad \Rightarrow \quad T \sim \mathrm{T}(n-1)$$

Método da região crítica:

$P\left(\overline{X} < c \mid \mu = \mu_0\right) = \alpha$	$P\left(\overline{X} > c \mid \mu = \mu_0\right) = \alpha$	$P\left(\overline{X} < c_1 \mid \mu = \mu_0\right) = P\left(\overline{X} > c_2 \mid \mu = \mu_0\right) = \frac{\alpha}{2}$
$RC = \left\{\overline{X} : \overline{X} < c\right\}$	$RC = \left\{\overline{X} : \overline{X} > c\right\}$	$RC = \left\{\overline{X} : \overline{X} < c_1 \vee \overline{X} > c_2\right\}$

$\bar{x} \in \mathrm{RC} \quad \Rightarrow \quad$ Rejeita H_0

Método do valor de prova:

$Valor\ p = P\left(\overline{X} \leq \bar{x} \mid \mu = \mu_0\right)$	$Valor\ p = P\left(\overline{X} \geq \bar{x} \mid \mu = \mu_0\right)$	$Valor\ p = 2\min\left\{P\left(\overline{X} \leq \bar{x} \mid \mu = \mu_0\right), P\left(\overline{X} \geq \bar{x} \mid \mu = \mu_0\right)\right\}$

$Valor\ p < \alpha \quad \Rightarrow \quad$ Rejeita H_0

Teste de hipóteses para μ₁-μ₂

$\mathrm{H}_0: \mu_1 - \mu_2 = k$ $\mathrm{H}_1: \mu_1 - \mu_2 < k$	$\mathrm{H}_0: \mu_1 - \mu_2 = k$ $\mathrm{H}_1: \mu_1 - \mu_2 > k$	$\mathrm{H}_0: \mu_1 - \mu_2 = k$ $\mathrm{H}_1: \mu_1 - \mu_2 \neq k$

σ_1 e σ_2 conhecidos

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\left[\begin{cases} n_1 \geq 30 \\ n_2 \geq 30 \end{cases} \vee \begin{cases} X_1 \sim \mathrm{N}(\mu_1, \sigma_1^2) \\ X_2 \sim \mathrm{N}(\mu_2, \sigma_2^2) \end{cases}\right] \Rightarrow Z \sim \mathrm{N}(0,1)$$

σ_1 e σ_2 desconhecidos

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$\sigma_1 = \sigma_2 = \sigma$ desconhecido

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$\begin{cases} n_1 \geq 30 \\ n_2 \geq 30 \end{cases} \Rightarrow T \sim \mathrm{N}(0,1)$$

$$\begin{cases} n_1 < 30 \vee n_2 < 30 \\ X_1 \sim \mathrm{N}(\mu_1, \sigma_1^2) \\ X_2 \sim \mathrm{N}(\mu_2, \sigma_2^2) \end{cases} \Rightarrow \begin{cases} T = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim \mathrm{T}(m), \quad m \cong \dfrac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}} \\ T = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim \mathrm{T}(n_1 + n_2 - 2) \end{cases}$$

Método da região crítica :

$P(\bar{X}_1 - \bar{X}_2 < c \mid \mu_1 - \mu_2 = k) = \alpha$	$P(\bar{X}_1 - \bar{X}_2 > c \mid \mu_1 - \mu_2 = k) = \alpha$	$P(\bar{X}_1 - \bar{X}_2 < c_1 \mid \mu_1 - \mu_2 = k) = \frac{\alpha}{2}$ $P(\bar{X}_1 - \bar{X}_2 > c_2 \mid \mu_1 - \mu_2 = k) = \frac{\alpha}{2}$
$RC = \{\bar{X}_1 - \bar{X}_2 : \bar{X}_1 - \bar{X}_2 < c\}$	$RC = \{\bar{X}_1 - \bar{X}_2 : \bar{X}_1 - \bar{X}_2 > c\}$	$RC = \{\bar{X}_1 - \bar{X}_2 : \bar{X}_1 - \bar{X}_2 < c_1 \vee \bar{X}_1 - \bar{X}_2 > c_2\}$

$\bar{x}_1 - \bar{x}_2 \in \mathrm{RC} \Rightarrow$ Rejeita H_0

Teste de hipóteses para μ 1-μ 2

Método do valor de prova:

$Valor\ p = P(\bar{X}_1 - \bar{X}_2 \leq \bar{x}_1 - \bar{x}_2 | \mu_1 - \mu_2 = k)$ | $Valor\ p = P(\bar{X}_1 - \bar{X}_2 \geq \bar{x}_1 - \bar{x}_2 | \mu_1 - \mu_2 = k)$ | $Valor\ p = 2\min\{P(\bar{X}_1 - \bar{X}_2 \leq \bar{x}_1 - \bar{x}_2 | \mu_1 - \mu_2 = k), P(\bar{X}_1 - \bar{X}_2 \geq \bar{x}_1 - \bar{x}_2 | \mu_1 - \mu_2 = k)\}$

$Valor\ p < \alpha \quad \Rightarrow \quad \text{Rejeita } H_0$

Teste de hipóteses para p

$H_0 : p = p_0$, $H_1 : p < p_0$ | $H_0 : p = p_0$, $H_1 : p > p_0$ | $H_0 : p = p_0$, $H_1 : p \neq p_0$

$$Z = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$n \geq 30 \Rightarrow Z \sim N(0,1)$

Método da região crítica :

$P(\hat{P} < c \mid p = p_0) = \alpha$, $RC = \{\hat{P} : \hat{P} < c\}$ | $P(\hat{P} > c \mid p = p_0) = \alpha$, $RC = \{\hat{P} : \hat{P} > c\}$ | $P(\hat{P} < c_1 \mid p = p_0) = P(\hat{P} > c_2 \mid p = p_0) = \frac{\alpha}{2}$, $RC = \{\hat{P} : \hat{P} < c_1 \vee \hat{P} > c_2\}$

$\hat{p} \in \text{RC} \quad \Rightarrow \quad \text{Rejeita } H_0$

Método do valor de prova:

$Valor\ p = P(\hat{P} \leq \hat{p} \mid p = p_0)$ | $Valor\ p = P(\hat{P} \geq \hat{p} \mid p = p_0)$ | $Valor\ p = 2\min\{P(\hat{P} \leq \hat{p} \mid p = p_0), P(\hat{P} \geq \hat{p} \mid p = p_0)\}$

$Valor\ p < \alpha \quad \Rightarrow \quad \text{Rejeita } H_0$

Teste de hipóteses para p 1-p 2

$\mathrm{H_0}: p_1 - p_2 = k$ $\mathrm{H_1}: p_1 - p_2 < k$	$\mathrm{H_0}: p_1 - p_2 = k$ $\mathrm{H_1}: p_1 - p_2 > k$	$\mathrm{H_0}: p_1 - p_2 = k$ $\mathrm{H_1}: p_1 - p_2 \neq k$

$$p_1 \neq p_2 \quad \Rightarrow \quad Z = \frac{\left(\hat{P}_1 - \hat{P}_2\right) - (p_1 - p_2)}{\sqrt{\dfrac{\hat{P}_1\left(1-\hat{P}_1\right)}{n_1} + \dfrac{\hat{P}_2\left(1-\hat{P}_2\right)}{n_2}}} \qquad p_1 = p_2 \quad \Rightarrow \quad Z = \frac{\left(\hat{P}_1 - \hat{P}_2\right) - (p_1 - p_2)}{\sqrt{\hat{P}\left(1-\hat{P}\right)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}\ , \qquad \hat{P} = \frac{n_1\hat{P}_1 + n_2\hat{P}_2}{n_1 + n_2}$$

$$\left(n_1 \geq 30 \wedge n_2 \geq 30\right) \Rightarrow Z \sim N(0,1)$$

Método da região crítica :

$P\left(\hat{P}_1 - \hat{P}_2 < c \mid p_1 - p_2 = k\right) = \alpha$ $RC = \left\{\hat{P}_1 - \hat{P}_2 : \hat{P}_1 - \hat{P}_2 < c\right\}$	$P\left(\hat{P}_1 - \hat{P}_2 > c \mid p_1 - p_2 = k\right) = \alpha$ $RC = \left\{\hat{P}_1 - \hat{P}_2 : \hat{P}_1 - \hat{P}_2 > c\right\}$	$P\left(\hat{P}_1 - \hat{P}_2 < c_1 \mid p_1 - p_2 = k\right) = \frac{\alpha}{2}$ $P\left(\hat{P}_1 - \hat{P}_2 > c_2 \mid p_1 - p_2 = k\right) = \frac{\alpha}{2}$ $RC = \left\{\hat{P}_1 - \hat{P}_2 : \hat{P}_1 - \hat{P}_2 < c_1 \vee \hat{P}_1 - \hat{P}_2 > c_2\right\}$

$$\hat{p}_1 - \hat{p}_2 \in \mathrm{RC} \quad \Rightarrow \quad \text{Rejeita } \mathrm{H_0}$$

Método do valor de prova:

$Valor\ p = P\left(\hat{P}_1 - \hat{P}_2 \leq \hat{p}_1 - \hat{p}_2 \mid p_1 - p_2 = k\right)$	$Valor\ p = P\left(\hat{P}_1 - \hat{P}_2 \geq \hat{p}_1 - \hat{p}_2 \mid p_1 - p_2 = k\right)$	$Valor\ p = 2\min\left\{P\left(\hat{P}_1 - \hat{P}_2 \leq \hat{p}_1 - \hat{p}_2 \mid p_1 - p_2 = k\right), P\left(\hat{P}_1 - \hat{P}_2 \geq \hat{p}_1 - \hat{p}_2 \mid p_1 - p_2 = k\right)\right\}$

$$Valor\ p < \alpha \quad \Rightarrow \quad \text{Rejeita } \mathrm{H_0}$$

Tipos de erro nos testes de hipóteses

$$\alpha = P(\text{Erro do tipo I}) = P(\text{Rejeitar } \mathrm{H_0} \mid \mathrm{H_0} \text{ verdadeira})$$

$$\beta = P(\text{Erro do tipo II}) = P(\text{Não rejeirar } \mathrm{H_0} \mid \mathrm{H_0} \text{ falsa})$$